$$pv = R\theta \qquad (22)$$

Then for the integrating factor μ_0 we can set according to equation (11) that

$$\frac{1}{\mu_0} = R\theta;\ \mu_0 = \frac{1}{R\theta} \qquad (23)$$

Again, knowing that for a perfect gas $\frac{\partial U}{\partial v}$ in (21) would be equal to zero we have

$$\frac{\partial U}{\partial v} = 0;\ \frac{\partial U}{\partial \theta} = C_v = \alpha \qquad (24)$$

Therefore by (21) and (22)

$$-\alpha \cdot \left(\frac{\partial \theta}{\partial v}\right)_M = \frac{R\theta}{v} \qquad (25)$$

The integral of (25) is consequently

$$\theta \cdot v^{\frac{R}{\alpha}} = f(M) \qquad (26)$$

and for the simplest form of $f(M) = M$ we have

$$\mu = \theta \cdot v^{\frac{R}{\alpha}} \cdot \frac{1}{R\theta} = \frac{1}{R} \cdot v^{\frac{R}{\alpha}} \qquad (27)$$

In other words a further integrating factor is a function of v only whereas the original one was $\mu = \frac{1}{\theta}$.

Multiplying the Energy Equation with the μ of (27) we have for an ideal gas in view of (24) and (22) that

$$R_\mu\, dQ = v^{\frac{R}{\alpha}} \cdot \frac{\partial U}{\partial \theta} \cdot d\theta + v^{\frac{R}{\alpha}} \cdot p \cdot dv$$

$$= v^{\frac{R}{\alpha}} \cdot \alpha \cdot d\theta + R\theta \cdot v^{\frac{R}{\alpha}-1} \cdot dv \qquad (28)$$

The latter is a perfect differential since we have

$$\frac{\partial}{\partial v}\left(\alpha\, v^{\frac{R}{\alpha}}\right) = \frac{\partial}{\partial \theta}\left(R\theta\, v^{\frac{R}{\alpha}-1}\right) \qquad (29)$$

Elsewhere I have shown that the particular integrating factor given by

$$\left.\begin{aligned} pV &= \frac{1}{\mu} \\ \text{with } \mu &= f\,(U) \end{aligned}\right\} \qquad (30)$$

leads to a consideration of a new type of cycle analogous to that of Carnot. The new cycle comprises two adiabatics and two iso-U curves instead of two isothermals.

CRYSTALLOGRAPHY.—*The crystallography and optical properties of β-lactose.*[1] EDGAR T. WHERRY, Bureau of Chemistry and Soils.

Although the crystallographic features of ordinary α-lactose have been fully described, there appear to be no data on the β-form. In the study of the development of minute crystals of sugars in ice cream, the Bureau of Dairy Industry of the United States Department of Agriculture found it desirable to have means for distinguishing these two forms of lactose from one another as well as from sucrose, and the examination of the grains by the immersion method under the polarizing microscope seems well adapted to the purpose. Accordingly O. E. Williams of that Bureau prepared and turned over to the writer a sample of crystallized β-lactose, in order that its properties might be determined and contrasted with those of other sugars. The crystals were obtained by holding a concentrated lactose solution at a temperature of about 94°C. They were then washed several times with hot glycerol and hot ethanol. The melting point was found to be 252.4°C.; since the melting point given in the literature is 252.2°, their identity was thus confirmed.

The crystals, which range from 1 to 5 mm. in diameter, are transparent and colorless. They have a pronounced polar development, and the distribution of their faces show that they belong to the holoaxial-polar (sphenoidal) class of the monoclinic system. Measurements of 10 crystals were made on the two-circle goniometer, with the results presented in Table 1.

[1] Received March 19, 1928.